TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 729

THE FLIGHT OF AN AUTOGIRO AT HIGH SPEED

By J. A. J. Bennett

Zeitschrift für Flugtechnik und Motorluftschiffahrt
Vol. 24, No. 17, September 14, 1933

1.6.1
1.7.3.1
1.5.2.9

Washington
December 1933

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 729

THE FLIGHT OF AN AUTOGIRO AT HIGH SPEED*

By J. A. J. Bennett

I. NOTATION

v, forward speed of autogiro.

w, mean induced velocity through the disk.

r, radial distance of blade element dr from rotor shaft.

R, extreme radius.

n, number of blades.

t, blade width; for simplicity, considered as constant from root to tip.

α_g, blade angle, equally assumed constant, measured from "zero-lift" position.

ψ, angular position of blade, measured from position conformably to figure 2.

ω, angular velocity of blades.

D, r.p.m. of rotor.

σ, "solidity," i.e., ratio of total blade area to rotor disk area $= \frac{nt}{(\pi R)}$.

ρ, air density (standard value at sea level, that is, "newton"/m³ or kg s²/m⁴).

c_{w_m}, mean profile drag coefficient for the employed profile.

*"Über den Flug eines Autogiro mit grosser Geschwindigkeit." Z.F.M., September 14, 1933, pp. 465-470.

α, angle of attack of autogiro, i.e., angle of rotor shaft to the vertical of the flight speed, counted positive when shaft is rearward inclined.

β, flapping angle, i.e., angle between blade and plane perpendicular to rotor shaft.

β_0, coning angle, i.e., angle between blade and its median plane of rotation.

β_1, angle of median plane of rotation of blades to plane perpendicular to rotor shaft.

W, total drag of rotor.

W_1, induced drag of rotor.

W_2, profile drag of rotor.

W_3, structural drag in kg at 161 km/h (100 mi./hr.).

A, lift.

H, thrust of rotor.

H_c, $\dfrac{H}{\pi\ \rho\ \omega^2\ R^4}$, i.e., load factor.

k_a, $\dfrac{A}{\pi\ R^2\ \rho\ v^2}$, i.e., second type of load factor.

λ, $\dfrac{v}{\omega\ R}$, i.e., a kind of speed ratio.

u, mean velocity through disk parallel to rotor shaft.

x, $u/(\omega R)$.

i, load rating of disk $= \dfrac{H}{\pi\ R^2}$

ξ, $\sqrt{r^2 + \dfrac{v^2}{\omega^2} + 2r\ \dfrac{v}{\omega}\ \sin\psi}$.

ηP, power required to overcome rotor drag.

ηP_a, power available (hp.).

II. INTRODUCTION

The modern autogiro consists essentially of a conventional engine, propeller, and fuselage supported by freely rotating surfaces. The power required to overcome the drag of the rotating wings is - with the exception of that necessary for imparting an initial rotation at take-off - not supplied by the engine but by the air flowing upward through the rotor disk. When the profile drag is sufficiently low the upward air motion through a portion of the rotor disk operating as a windmill produces a higher torque than absorbed by the drag; the remainder of the blade therefore operates as propeller (reference 1). In contrast to the orthodox airplane on which the wings move through the air at the same speed as the fuselage, the autogiro can fly at very low forward speed (or at zero forward speed in vertical descent) while its rotor blades move at high speed.

By virtue of horizontal and vertical hinges at the hub, the blades are free to move up and down and to change their neutral angular position to each other to a certain degree. As a result the blades assume, during flight, a position relative to the hub, so that the resultant of the forces on the blades passes through the center of the hinge joint. The blades themselves are raised when revolving in flight direction and lowered when rotating in the opposite direction.

The principal component of the wind force on each blade, i.e., the lift, is balanced by the centrifugal force component and the blades take up a coning position. The "coning angle" β_0, is usually very small and can be disregarded in this analysis. The "flapping angle" β can be expanded as a Fourier series in the form ψ, the momentary angular blade setting. If the centrifugal force is great compared to the air forces, all terms of the Fourier expansion save the first one, can be neglected, so that

$$\beta \approx \beta_1 \cos (\psi - \psi_1)$$

where $\psi_1 = 0$ under the above assumption and when disregarding the gravity effect on the blades. So in the first approximation the up and down flapping of the blades is equivalent to a rotation of the plane of rotation through an angle β_1 (fig. 1).

Lock has shown that, by selection of the plane of motion of the blades as reference plane, the flapping of the blades can be disregarded; that in this reference system the blades have, while revolving, a periodically changing blade angle. Consequently, the autogiro is mechanically equivalent to an aircraft with fixed wings, on which the blade angle of the wings is periodically changed by means of some special device and whereby the axis of rotation is inclined at angle β_1 to the original axis.

The calculation of angle β_1 results from the consideration of the thrust moment of a wing about the hinge which, according to the previous assumption, is independent of ψ.

Glauert (reference 2) disregarded the squares and higher powers of the tip-speed ratio λ in his analysis of the flow at a blade element and so underestimated the power of an autogiro at high speed. Lock (reference 3) repeated this calculation without omitting important terms and found that Glauert's theory must be so much more modified as v and λ increase. The modified theory gives the same terms for the profile drag as Glauert's in appendix I, of his report, that is,

$$\frac{W_2}{A} = \frac{c_{w_m}\,\sigma}{8\,\lambda\,H_c}\,(1 + 3\,\lambda^2)$$

with consideration to the energy loss.

In section IV, it is shown that this equation becomes

$$\frac{W_2}{A} = \frac{c_{w_m}\,\sigma}{8\,\lambda\,H_c}\,(1 + 4.65\,\lambda^2),$$

when the effect of the radial velocity is taken into account.

The energy balance of an autogiro rests on the fact that its power (Wv) must be equivalent to the power required for overcoming the profile drag of the rotor blades plus the power necessary to produce the vertical velocity w. If the torque were other than zero, yet another term for the energy equation would have to be added.

A simple method of allowing for the radial velocity is obtained by the hitherto overlooked fact that the direction of the resultant effect of the wind on the rotor is coincident with the axis of the coning surface described by the blades while revolving.

According to figure 1,

$$W = H\ (\alpha + \beta_1).$$

This equation is used in section IV to tie α and β to the fundamental quantities. Glauert and Lock's assumption of constant axial velocity through the rotor disk has been retained. Although the velocity near the points $r = \frac{v}{\omega}$, $\psi = \frac{3}{2}\pi$ (fig. 2) is high, it is low across the effective range of the blade and patently negative at the blade tip (reference 1). The mean velocity u through the disk being small, it may with sufficient accuracy be presumed that $x = \frac{u}{\omega R}$ and that at low angles of attack the distribution of the induced velocity is similar to that of a fixed-wing system, i.e., that a so-called Lanchester-Prandtl vortex system is formed.

The contention that in flying a stated distance the rotating wing describes a much longer path than the fixed wing of a conventional airplane, and that for this very reason the energy loss must be greater, is not plausible because the surface chord of the rotating wing is considerably less than that of the corresponding fixed wing. The induced power cannot, by virtue of the similarity of the vortex system, be materially different from that of the conventional airplane of span equal to rotor diameter. The power loss due to profile drag is, to be sure, greater on account of the higher relative speed of the blades per unit surface, but by assuming sufficiently small solidity the total power loss can even become less than on the corresponding fixed-wing system or, in other words, the maximum speed of the autogiro can actually be higher than that of the corresponding airplane and that without sacrificing the advantage of low landing speed.

Hitherto the autogiro had a high structural drag, due in part to the large tail surfaces necessary to insure the initial rotor rotation by deflecting the slipstream; then, to the elastic bracing of the wings and the existence of a small fixed wing carrying the ailerons. On the latest autogiro the rotor is coupled to the engine, the wings are cantilever, ailerons and elevator are omitted, the control being effected by tilting the rotor shaft.

Hereinafter follows a method for computing the flight performance of an autogiro at high speed, the velocity component along the blades being accounted for by calculation of the profile drag and the equation for zero torque.

III. INDUCED VELOCITY

Analysis of the air disturbances left behind reveals that the mean induced velocity w through the rotor disk of an autogiro of radius R at small angles of attack is approximately equal to the induced velocity of an airfoil of span $2R$ with equal total lift. Assuming that the number of blades is not too small, the strength and distribution of the produced vortices are sensibly the same as on the airfoil of span $2R$. On it the velocity w with elliptic lift distribution is

$$w = \frac{H}{2\,\pi\,R^2\,\rho\,v} \tag{1}$$

The discrepancy of this formula becomes larger as the number of blades becomes less, but this fact can be disregarded in the theory of the terms of the first order, because at small angles of attack the induced drag is much lower than the profile drag.

IV. EQUATION FOR ZERO TORQUE

The total reaction of the wind on the rotor is coincident with the effective axis of the cone described by the revolving blades. Thus the energy output per second is

$$W\,v = v\,H\,(\alpha + \beta_1).$$

Since, however, the torque is zero and the raising of the blades requires no energy, the total expended energy equals the amount of energy per second performed by the production of the profile and the induced drag. Therefore,

$$\begin{aligned} Wv = vH\,(\alpha + \beta_1) &= W_1 v + W_2 v \\ &= wH + \frac{n}{4\pi}\int_0^{2\pi} d\psi \int_0^R \rho\;t\;c_{w_m}\,\xi^3\,\omega^3\,dr \end{aligned} \tag{2}$$

whereby

$$\xi^2 \omega^2 = r^2 \omega^2 + v^2 + 2 v r \omega \sin \psi \tag{3}$$

$\xi\omega$ is the resultant of flight speed v and tip speed $r\omega$ of blade element dr at distance r from the rotor shaft. It is equivalent to a pure rotation at angular speed ω about a point defined (see fig. 2) by the polar coordinate

$$r = \frac{v}{\omega}, \quad \psi = \frac{3\pi}{2}$$

with $\Xi^2 \omega^2 = R^2 \omega^2 + v^2 + 2v R \omega \sin \psi$,

the second term on the right-hand side of equation (2) becomes

$$W_2 v = \frac{n \rho t c_{w_m}}{4\pi} \int_0^{2\pi} d\psi \int_{\frac{v}{\omega}}^{\Xi} \frac{\xi^2 \omega^3 d\xi}{\sqrt{1 - \left(\frac{v \cos \psi}{\xi \omega}\right)^2}}$$

In addition, since $\frac{v \cos \psi}{\xi \omega}$ is less than 1 over the major part of the disks, we can expand

$$\left[1 - \left(\frac{v \cos \psi}{\xi \omega}\right)^2\right]^{-1/2}$$

to a series

$$1 + \frac{1}{2}\left(\frac{v \cos \psi}{\xi \omega}\right)^2 + \frac{3}{8}\left(\frac{v \cos \psi}{\xi \omega}\right)^4 + \frac{5}{16}\left(\frac{v \cos \psi}{\xi \omega}\right)^6 + \text{etc.}$$

This series converges quickly.

Limited to the first three terms, the integration yields

$W_2 v$

$$= \frac{R^5 \sigma \rho c_{w_m} \omega^3 \pi}{8} \left[1 + \frac{9}{2} \lambda^2 + \frac{3}{4}\left(\log \frac{1}{\lambda}\right) \lambda^4 + \frac{3}{16} \lambda^6 - \frac{3}{128} \lambda^8\right]$$

$$= \frac{R^5 \sigma \rho c_{w_m} \omega^3 \pi}{8} \left[1 + m \lambda^2\right],$$

m being given in the appended tabulation:

λ	1.0	0.75	0.60	0.50	0.40	0.30	0.00
m	4.67	4.67	4.66	4.64	4.61	4.58	4.50

Taking m = 4.65 for small angles of attack, equation (2) gives

$$\frac{W}{A} = \alpha + \beta_1 = \frac{w}{v} + \frac{\sigma\, c_{w_m}}{8\, H_c\, \lambda}\,(1 + 4.65\,\lambda^2),$$

whereby

$$H_c = \frac{H}{\pi\, \rho\, \omega^2\, R^4}$$

or, since according to (1)

$$\frac{w}{v} = \frac{H}{2\,\rho\, v^2\, \pi\, R^2},$$

$$\frac{W}{A} = \alpha + \beta_1 = \frac{H_c}{2\,\lambda^2} + \frac{\sigma\, c_{w_m}}{8\, H_c\, \lambda}\,(1 + 4.65\,\lambda^2) \qquad (4)$$

This is the equation for zero torque.

V. FUNDAMENTAL EQUATIONS

Since the radial velocity component can be neglected when computing the lift and β_1, we have, according to Lock (reference 3)

$$\beta_1 = \frac{2\,\lambda\left(x + \frac{4}{3}\,\alpha_g\right)}{1 - \frac{1}{2}\,\lambda^2} \qquad (5)$$

and

$$H_c = \sigma\left[\frac{3}{2}\,x + \alpha_g\left(1 + \frac{3}{2}\,\lambda^2\right)\right] \qquad (6)$$

with

$$x = \lambda\left(\alpha - \frac{w}{v}\right) = \lambda\left(\alpha - \frac{H_c}{2\,\lambda^2}\right) \qquad (7)$$

Equations (4), (5), and (7) give

$$\frac{x}{\lambda} + \frac{2\lambda\left(x + \frac{4}{3}\alpha_g\right)}{1 - \frac{1}{2}\lambda^2} = \frac{\sigma\, c_{w_m}}{8\, H_c\, \lambda}\,(1 + 4.65\,\lambda^2)$$

or

$$x\left(1 + \frac{3}{2}\lambda^2\right) + \frac{8}{3}\alpha_g\,\lambda^2 = \frac{\sigma\, c_{w_m}}{8\, H_c}\,(1 + 4.65\,\lambda^2)\left(1 - \frac{1}{2}\lambda^2\right).$$

According to (6)

$$x = \frac{2}{3}\left[\frac{H_c}{\sigma} - \alpha_g\left(1 + \frac{3}{2}\lambda^2\right)\right], \qquad (8)$$

hence

$$\frac{H_c}{\sigma}\left(1 + \frac{3}{2}\lambda^2\right) - \alpha_g\left(1 + \frac{3}{2}\lambda^2\right)^2 + 4\,\alpha_g\,\lambda^2 = \frac{3\,\sigma\, c_{w_m}}{16\, H_c}\,(1 + 4.65\,\lambda^2)\left(1 - \frac{1}{2}\lambda^2\right)$$

or

$$\left(\frac{H_c}{\sigma}\right)^2 + \left[\frac{4\,\lambda^2}{1 + \frac{3}{2}\lambda^2} - \left(1 + \frac{3}{2}\lambda^2\right)\right]\alpha_g\left(\frac{H_c}{\sigma}\right) = \frac{3}{16}\, c_{w_m}\,\frac{(1 + 4.65\,\lambda^2)\left(1 - \frac{1}{2}\lambda^2\right)}{1 + \frac{3}{2}\lambda^2} \qquad (9)$$

$\frac{H_c}{\sigma}$ being expressed by α_g, c_{w_m}, and λ.

VI. NUMERICAL RESULTS

Having defined H_c/σ for stated values of λ, α_g, and c_{w_m}, we compute x from (8). k_a is assessed for a value of σ from

$$k_a = \frac{H_c}{\lambda^2} = \left(\frac{H_c}{\sigma}\right)\frac{\sigma}{\lambda^2} \qquad (10)$$

and W/A from (4) in the form

$$\frac{W}{A} = \frac{k_a}{2} + \frac{c_{w_m}\sigma}{8\lambda^3 k_a}(1 + 4.65\lambda^2) \tag{11}$$

α can be determined from (7):

$$\alpha = \frac{x}{\lambda} + \frac{k_a}{2} \tag{12}$$

and β_1 from

$$\beta_1 = \frac{W}{A} - \alpha \tag{13}$$

The tip velocity ωR is obtained for varying values of the load rating i from the equation for H_c, namely,

$$\omega R = \sqrt{\frac{i}{\rho\, H_c}} \tag{14}$$

and v is defined from

$$v = \lambda\, \omega\, R \tag{15}$$

The rotor speed can be determined for a stated R from

$$D = \frac{60\,\omega}{2\pi} = 30\,\frac{\omega R}{\pi R} \tag{16}$$

Figures 3, 4, and 5 illustrate the effect of solidity, blade angle, and profile drag on W/A for values of k_a less than 0.10.

VII. FORWARD SPEED

To insure suitable landing qualities and slow rate of descent (reference 1), it is necessary to limit the disk loading i. To illustrate: Assume $i = 9.76$ kg/m² (2 lb./sq.ft.). According to (14) and (15),

$$\omega R = 8.83/\sqrt{H_c}$$

$$v = 8.83\,\lambda/\sqrt{H_c}.$$

Figures 6, 7, and 8 show the effect of the solidity, the blade angle, and the profile drag on W/A for a number of v values.

The results indicate that for high speed, σ should be as small as possible, and α_g be as large as possible for a given c_{w_m}. It is therefore of advantage to use thin, narrow blades, on which the flow breaks down gradually. When α_g is too great, the flow breaks down on the retreating blade and c_{w_m} becomes excessively great.

VIII. MAXIMUM SPEED

The power ηP necessary to overcome the rotor drag is given by

$$\frac{75\ \eta P}{A} = \left(\frac{W}{A}\right) v \tag{17}$$

The effect of σ, α_g, and i on the $\eta P/A$ is shown in figures 9, 10, and 11 for divers values v.

If W_3 is the structural drag (in kilograms) at 161 km/h (100 mi./hr.) speed, the thrust required to overcome the structural drag for v km/h is:

$$W_3\ v^3 \times 1.43 \times 10^{-7}$$

Then the maximum speed v_m is given by

$$\eta P_a = \eta P + (W_3\ v_m{}^3 \times 1.43 \times 10^{-7}) \tag{18}$$

ηP_a = total available thrust power.

IX. COMPARISON WITH AN AIRPLANE

W/A changes but little with v at high speed, hence the power loading $\eta P/A$ increases sensibly proportional to v, as seen from equation (17). On an airplane with span equivalent to the diameter of the autogiro and with the same structural drag, the power loading is practically proportional to v^3. Consequently, the autogiro can attain a higher maximum speed than an airplane of the same power loading (reference 4).

Th. v. Karman published a comparison of flight performances on an autogiro, an orthodox airplane, and a helicopter (reference 5), but his autogiro evinced a high structural drag. To compare the performance of an autogiro with that of a conventional airplane, we designate an airplane as "comparable" whose span equals the rotor diameter of the autogiro and which has the same lift and structural drag and the same thrust horsepower, and the same wing profile.

Let W_2' = profile drag,

c_w' = drag coefficient of profile, and

S = aspect ratio of the airplane.

Then,
$$W_2' = c_w' \rho v^2 \frac{2 R^2}{S} \tag{19}$$

From (4) follows

$$W_2 = c_{w_m} \rho v^2 (1 + 4.65 \lambda^2) \frac{\pi R^2 \sigma}{8 \lambda^3}$$

hence
$$\frac{W_2}{W_2'} = \frac{\pi \sigma S}{16 \lambda^3} (1 + 4.65 \lambda^2) \tag{20}$$

If the area of the wings is the same as that of the autogiro blades, it gives:

$$\frac{4 R^2}{S} = \sigma \pi R^2 .$$

Consequently,

$$\frac{W_2}{W_2'} = \left(\frac{c_{w_m}}{c_w'}\right) \frac{(1 + 4.65 \lambda^2)}{(4 \lambda^3)} > 1,$$

when $\lambda < 1$.

An airplane with the same wing area as the modern autogiro would, however, have a very high landing speed, and therefore cannot be considered as a "comparative airplane." For autogiro blades of Göttingen section 429 and $\sigma = 0.07$ and $i = 9.76$, the surface loading is $i/\sigma = 139.3$ kg/m² (28.5 lb./sq.ft.). The maximum lift coefficient of airfoil Göttingen 429 is about 1.16, and the landing speed = $144 \sqrt{\frac{139.3}{1.16}} = 158$ km/h (98.2 mi./hr.). The rigid surface of the "comparative airplane" must for that reason be greater than the autogiro blades. With the landing speed of the "comparative airplane" limited to 80.5 km/h (50.0 mi./hr.), the wing loading is 36.1 kg/m² (7.4 lb./sq.ft.), and the wing area then 3.86 times as great as the blade area of the autogiro.

Figure 12 shows the performance of an autogiro with the parameters $\alpha_g = 4^\circ = 0.07$; $\sigma = 0.07$; $c_{w_m} = 0.012$; $i = 9.76$ in contrast to our defined "comparative airplane." Equation (19) becomes

$$\frac{W_2'}{A} = \frac{c_w' \rho v^2}{72.2} = 0.173 \left(\frac{v}{100}\right)^2$$

because $c_w' = 0.01$ and $\rho = 0.125$.

The thrust power $\eta P'$ for overcoming induced drag W_1' and profile drag W_2' is given with

$$\frac{75 \eta P'}{A} = \left(\frac{W_1'}{A} + \frac{W_2'}{A}\right) v$$

$$= \frac{A}{(2 \pi \rho R^2 v)} + 17.3 \left(\frac{v}{100}\right)^3$$

or

$$\frac{\eta P'}{A} = \frac{0.52}{v} + 0.231 \left(\frac{v}{100}\right)^3 . \qquad (21)$$

The $\eta P'/A$ values computed from this formula are included in figure 12.

X. REMARKS

As in Lock's previous report, the angle between the blade and its mean plane of rotation, has been disregarded, the axial speed component over the autogiro disk assumed to have a constant value, and the partial reversal of the trailing edge in the leading edge on the receding blade, neglected. As a result of which the calculation becomes so much more uncertain as the values of λ become greater.

On the high-speed autogiro of the future, with a ratio of over 7 of maximum to minimum horizontal speed, λ will range between 0.7 and 1.0. The efficiency within this range will probably be considerably lower than predicted in this analysis.

The writer wishes to express his appreciation to Mr. de la Cierva, Professor Prandtl, and Professor Betz, Gottingen, as well as to Professor Wieselsberger, Aachen, for their interest and support.

REFERENCES

1. Bennett, J. A. J.: Vertical Descent of the Autogiro. T.M. No. 673, N.A.C.A., 1932.

2. Glauert, H.: A General Theory of the Autogyro. R. & M. No. 1111, British A.R.C., 1926.

3. Lock, C. N. H.: Further Development of Autogyro Theory. Parts I and II. R. & M. No. 1127, British A.R.C., 1927.

4. Cierva, Juan de la: Jour. Roy. Aero. Soc., November 1930.

5. v. Kármán, Th.: Z.F.M., August 1931.

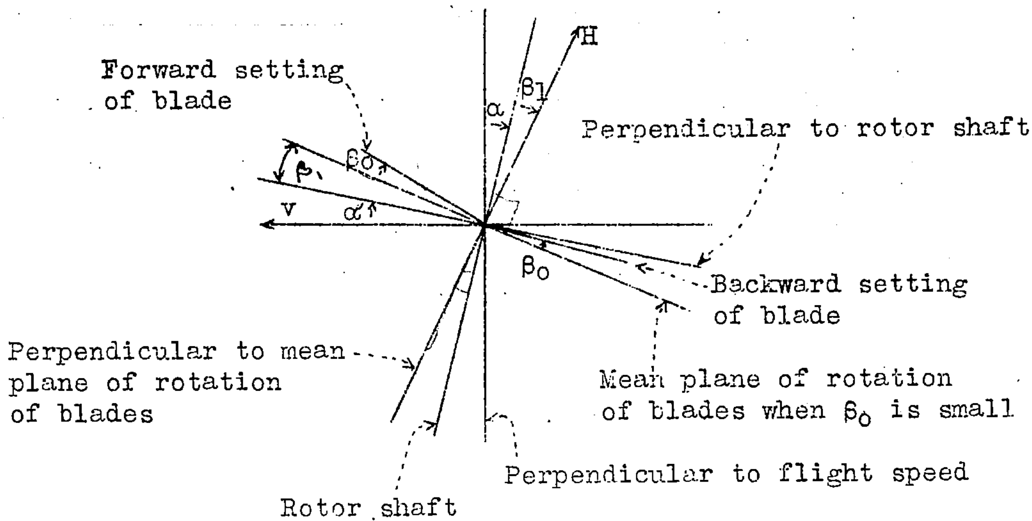

Figure 1

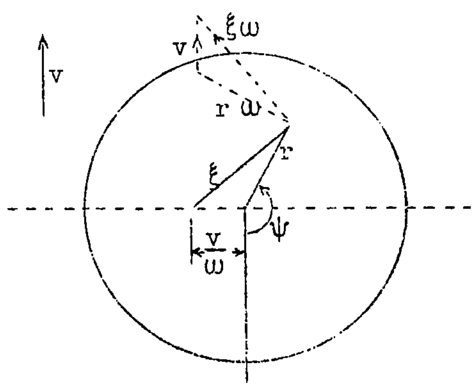

Figure 2

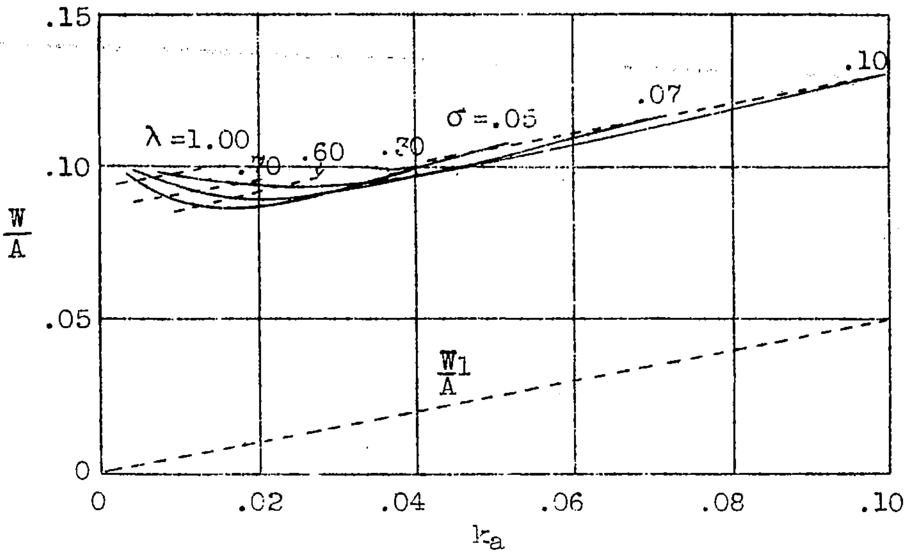

Figure 3.- Effect of solidity on W/A
$\alpha_g = 4^\circ = .07$; $c_{wm} = .012$

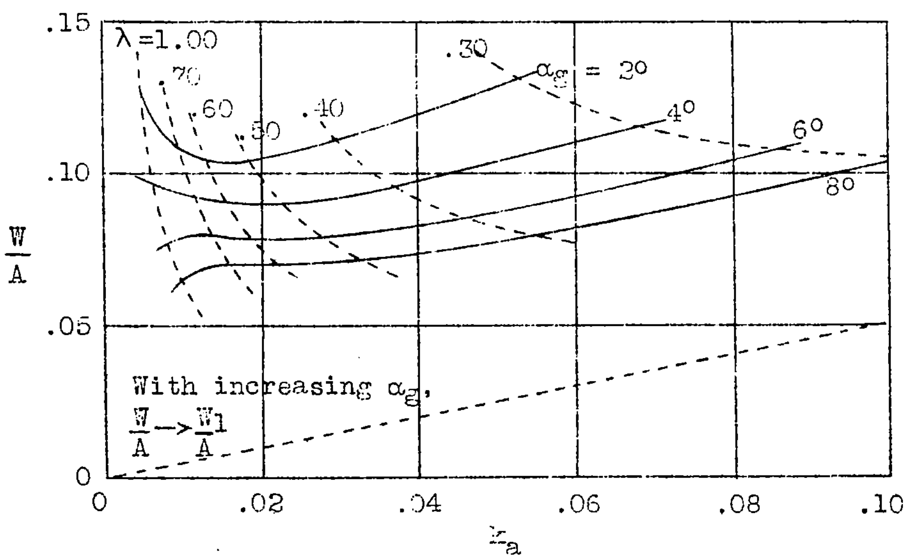

Figure 4.- Effect of blade angle on W/A
$\sigma = .07$; $c_{wm} = .012$

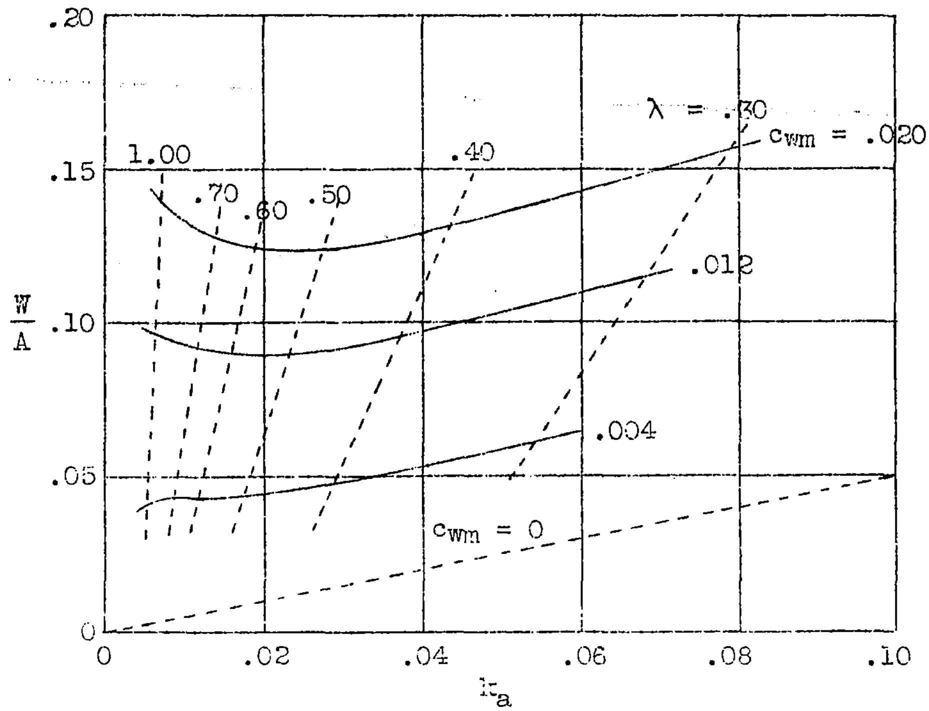

Figure 5.- Effect of profile drag on W/A
σ = .07; α_g = .07 = 4°

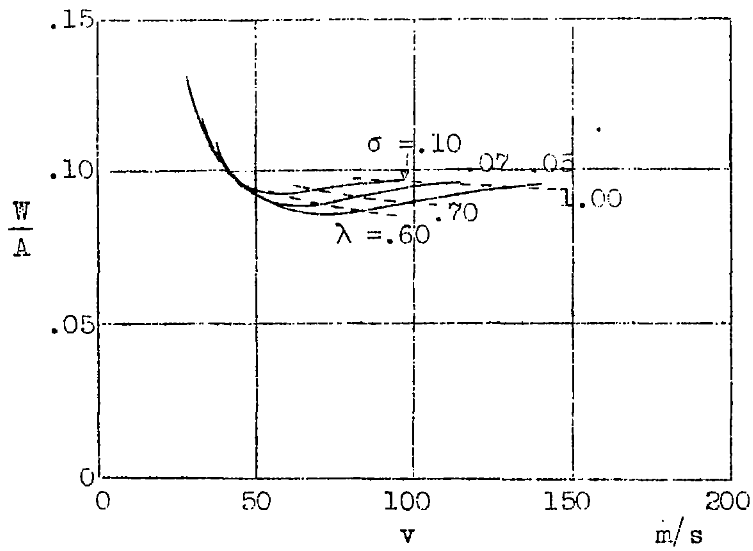

Figure 6.- Effect of solidity on W/A
α_g = 4° = .07; c_{wm} = .012; i = 9.76

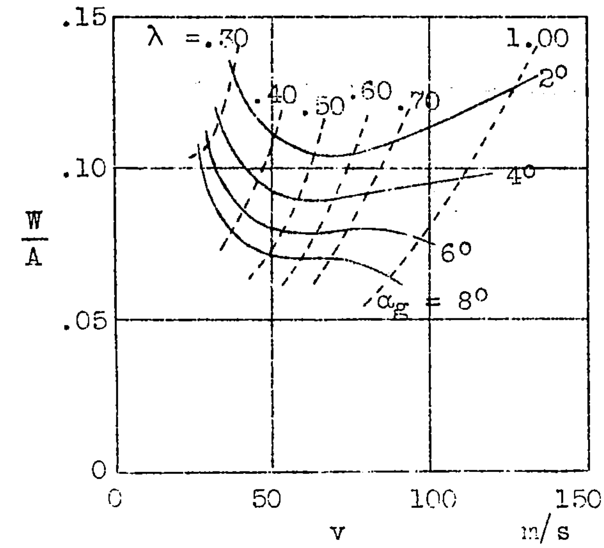

Figure 7.- Effect of blade angle on W/A
σ = .07; c_{wm} = .012; i = 9.76

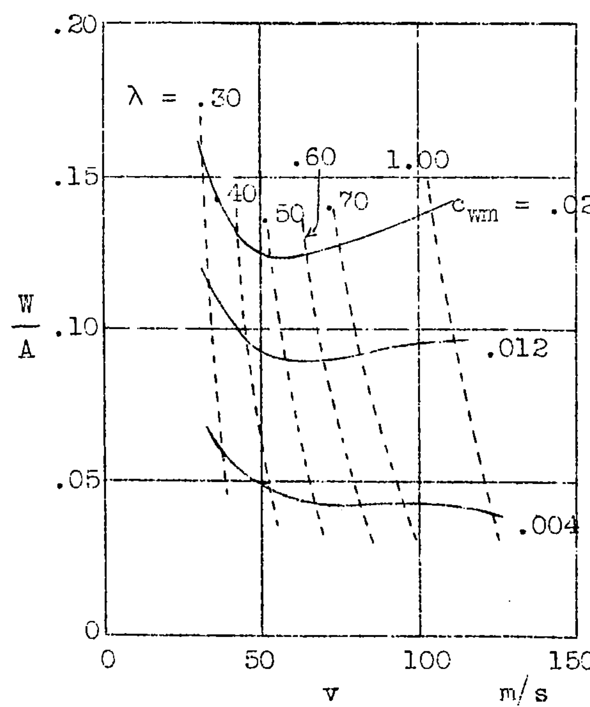

Figure 8.- Effect of profile drag on W/A
σ = .07; α_g = .07=4°; i = 9.76

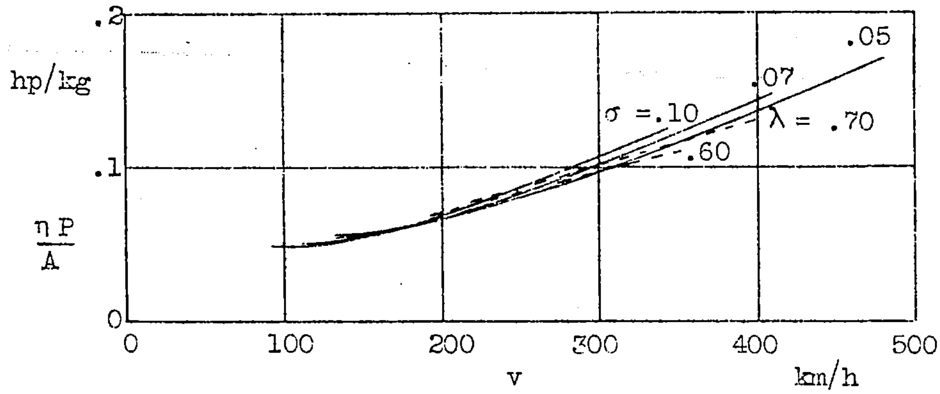

Figure 9.- Effect of solidity on $\eta P/A$
$\alpha_g = 4^\circ$ =.07; c_{wm} = .012; i = 9.76

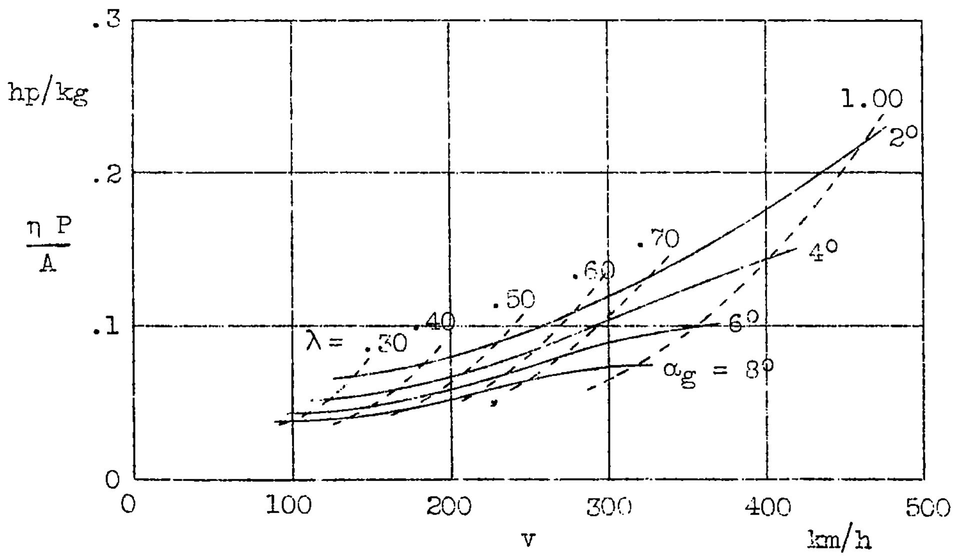

Figure 10.- Effect of blade angle on $\eta P/A$
σ = .07; c_{wm} = .012; i = 9.76

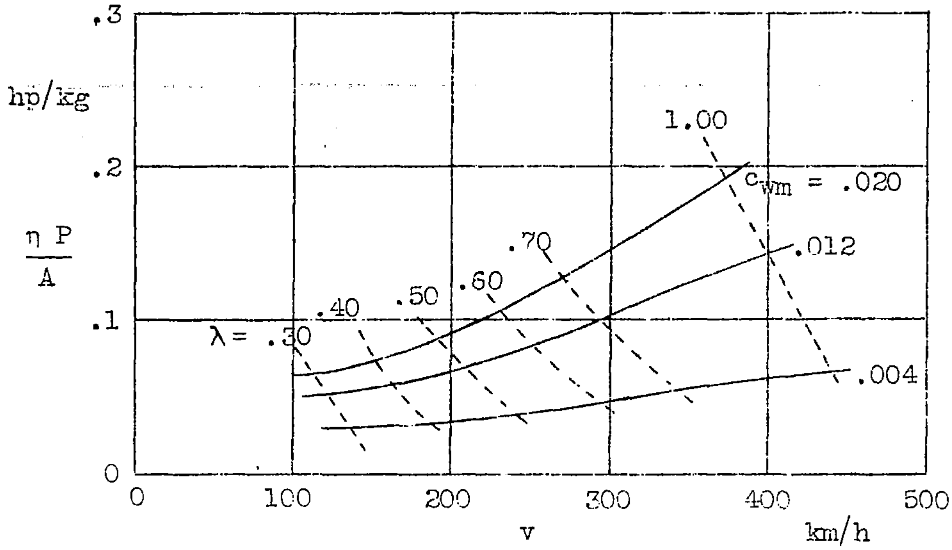

Figure 11.- Effect of profile drag on η P/A
σ = .07; α_g = .07 = 4°; i = 9.76

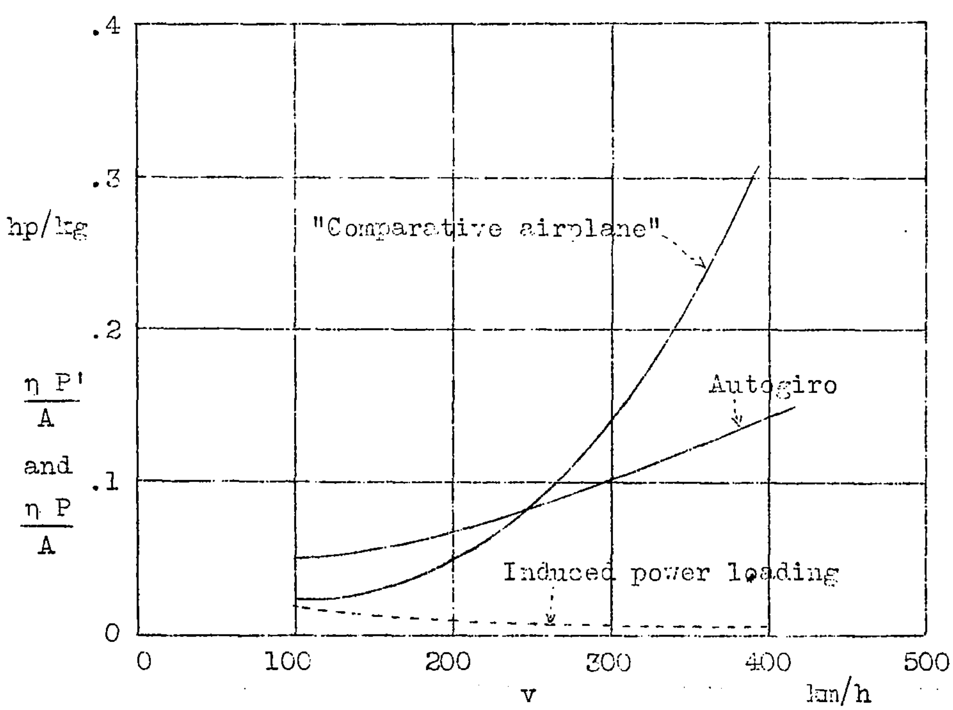

Figure 12.- Comparison between performance of an airplane and autogiro.
c_{wm} = .012, σ = .07, α_g = 4° i = 9.76